A Probabilistic Evaluation of Voltage Control Strategies in Active MV Networks

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Abstract—This paper attempts to thoroughly evaluate the long-term performance of the most well-established voltage control strategies applied in active medium-voltage (MV) networks. This is attained by performing an exhaustive probabilistic analysis to consider and assess the impact of generation and demand uncertainties on the network operation. More specifically, Monte Carlo-based time-series simulations are conducted to evaluate the long-term performance of the voltage control strategies in terms of overvoltage mitigation, minimization of network losses, and minimization of the overall reactive power consumption. These strategies are tested on an extended radial MV network with high distributed generation (DG) penetration, assuming loads and DG units follow specific distribution patterns.

Index Terms—Distributed power generation, loss minimization, probabilistic analysis, reactive power control, voltage control.

I. INTRODUCTION

On the way to sustainable energy, European countries have radically changed their energy policy towards the integration of distributed generation (DG), and especially of renewable energy sources (RES), in the power system. This was mainly initiated by the Directive 2009/28/EC, which foresees that by 2020, 20% of the consumed energy must be produced by RES units [1]. Furthermore, according to the draft proposal of [2], this percentage will be increased up to 27% by 2030, indicating the continuous increase of RES share even during the next decade. Consequently, there is a strong need to facilitate the transition towards the sustainable energy by effectively overcoming the problems that may occur regarding the secure and reliable network operation.

Voltage rise is considered as the most common technical challenge [3], prohibiting the further increase of DG penetration in distribution networks [4]. Focusing on medium-voltage (MV) networks, the traditional solutions cannot effectively address this issue. More specifically, grid reinforcement is an expensive option for the distribution system operators (DSOs) [5], while the use of feeder capacitors and the on-load tap changer (OLTC) of the high-voltage/medium-voltage (HV/MV) transformer cannot effectively regulate the network voltages [6], [7]. On the contrary, the reactive power control (RPC) of DG units has been recently introduced to tackle overvoltages and it is considered as the most promising and effective solution against the active power curtailment techniques, due to the relatively low R/X ratio of the MV lines [8].

Several RPC methods have been proposed in the literature to address the voltage rise problem, which can be classified into two main categories: decentralized and centralized control schemes. In the former, control actions are taken by the DG units based only on local measurements [9], while in the latter, optimization techniques are employed to optimally dispatch the reactive power of the DG units [10]. Although these methods have been thoroughly presented and examined in the literature, their performance has been evaluated using only deterministic simulations. As a result, the stochastic behavior of generation and demand is not considered, introducing inaccuracies concerning the real-time operation of these control schemes.

Scope of this paper is to compare the most well-established decentralized and centralized RPC methods in terms of overvoltage mitigation, minimization of network losses, and minimization of the overall reactive power consumption, by performing a probabilistic analysis. This is achieved by conducting Monte Carlo-based time-series simulations, assuming specific distribution patterns to take into account consumption and generation uncertainties.

The remaining of this paper is organized as follows: Section II presents two decentralized control schemes which have been already integrated into several grid codes for the interconnection of distributed generation. Section III presents the mathematical formulation of the centralized algorithms, aiming at minimizing network losses. The system under study and the Monte Carlo-based time-series simulation results are presented in Sections IV and V, respectively. Finally, Section VI concludes the paper.

II. DECENTRALIZED CONTROL SCHEMES

Currently, countermeasures against voltage violations are foreseen in several grid codes for the DG interconnection [11]–[14]. These mainly consist of decentralized RPC methods, which can be divided into static and dynamic methods based on whether the operational set-points of the DG units
are constant or dynamically changed following a predefined rationale [15]. Operating DG units with constant power factor is an indicative example of static RPC method, while the use of droop characteristics is a dynamic RPC method, presenting an improved performance against the static methods [16]. In this paper, two types of droop characteristics are considered, namely the $Q(V)$ and the $\cos \phi(P)$ droop characteristics, presented in detail below.

### A. First Droop Characteristic ($\cos \phi = f(P)$)

This type of droop characteristic has been already incorporated into the German [11] and Italian [12] grid codes, while its graphical representation is depicted in Fig. 1. According to this method, the power factor of the DG unit is determined with respect to the active power injection. In particular, during low generation periods, the DG unit provides reactive power support to the grid, whereas, during high generation periods, it absorbs reactive power to partially compensate the active power injection, thus mitigating network overvoltages. The mathematical formulation of this method is described as follows:

$$
\cos \phi = \begin{cases} 
  pf_{\min}, & \text{if } 0 \leq p_{\text{inj}} < p_1 \\
  pf_{\min} + s_1(p_{\text{inj}} - p_1), & \text{if } p_1 \leq p_{\text{inj}} < p_2 \\
  -1 + s_2(p_{\text{inj}} - p_2), & \text{if } p_2 < p_{\text{inj}} \leq p_3 \\
  -pf_{\min}, & \text{if } p_3 \leq p_{\text{inj}} \leq 1 
\end{cases} 
$$

(1)

where $pf_{\min}$ and $p_{\text{inj}}$ is the minimum power factor and the injected active power of the DG unit referred to nominal conditions. Furthermore, $s_1$ and $s_2$ are the slope coefficients of the two linear regions, whereas $p_1$, $p_2$, and $p_3$ are the three active power thresholds, determining the four operation regions.

### B. Second Droop Characteristic ($Q = f(V)$)

A typical form of the $Q = f(V)$ droop characteristic is presented in Fig. 2, where it can be observed that the reactive power of the DG unit is determined with respect to the voltage at the point of common coupling (PCC). Scope of this method is twofold:

1) Maintain a zero reactive power for a relatively large voltage operational range. In this way, unnecessary reactive power consumption/production is avoided, leading to reduced network losses.

2) Participate in the voltage regulation process by absorbing/producing reactive power. This feature is activated only when the PCC voltage approaches the permissible voltage limits.

The $Q = f(V)$ droop characteristic is included in the Italian [12], UK [13], and North America [14] grid codes. Nevertheless, it is worth noticing these grid codes do not propose any specific $Q(V)$ droop characteristic, leaving this definition to the local grid connection requirements [17]. The reactive power of the DG units is calculated according to the following piecewise function:

$$
Q = \begin{cases} 
  Q_{\max}, & \text{if } V_{\text{PCC}} < V_1 \\
  Q_{\max} - s_3(V_{\text{PCC}} - V_1), & \text{if } V_1 \leq V_{\text{PCC}} < V_2 \\
  0, & \text{if } V_2 \leq V_{\text{PCC}} < V_3 \\
  -s_4(V_{\text{PCC}} - V_3), & \text{if } V_3 \leq V_{\text{PCC}} < V_4 \\
  -Q_{\max}, & \text{if } V_{\text{PCC}} > V_4 
\end{cases} 
$$

(2)

where $V_{\text{PCC}}$ is the PCC voltage and $Q_{\max}$ is the maximum permissible reactive power of the DG unit, calculated according the reactive power capability curve. $s_3$ and $s_4$ are the slope coefficients of the two linear regions, while $V_1$, $V_2$, $V_3$, and $V_4$ denote the voltage thresholds. Generally, $V_1$ and $V_4$ correspond to the minimum and maximum permissible voltage limits, as defined by the standard EN 50160 [18].

### III. CENTRALIZED ALGORITHMS

The main scope of the centralized control schemes is to ensure the safety and reliability of the network, while also optimizing the network operation [19]. Generally, the optimization objective focuses on the minimization network losses, attained by employing two different types of network elements, i.e., the DG units and the on-load tap changer (OLTC) of the HV/MV transformer. Therefore, this problem is formulated as a mixed-integer nonlinear optimization method, as follows:

$$
\min \sum_{i \in B} P_{\text{loss},i} 
$$

(3)

Eq. (3) is the objective function of the optimization problem, aiming at the minimization of network losses. Furthermore, $B$ denotes the set of network branches and $P_{\text{loss},i}$ is the active
power loss of the $i$-th branch. The power flow equations are expressed by:

$$V_i^2 = \{V_i^2_p + 2A_{b(i)} + |V_i|^2 \}
+ 4A_{b(i)}V_i^2_p - 4C_{b(i)}^2 \}^{1/2} \forall i \in N$$ (4)

where $N$ denotes the set of network nodes. $V_i$ and $V_{i,p}$ stand for the voltage magnitudes of the $i$-th node and of the previous adjacent node, respectively, while $b(i)$ stands for the upstream branch, with respect to the HV/MV transformer, connected to the $i$-th node. Coefficients $A_{b(i)}$ and $C_{b(i)}$ are calculated as follows:

$$A_i = D_iR_i + E_iX_i \forall i \in B$$ (5)

$$C_i = D_iX_i - E_iR_i \forall i \in B$$ (6)

where $R_i$ and $X_i$ are the resistance and the reactance of the $i$-th branch. $D_i$ and $E_i$ are the active and reactive power flowing through the $i$-th branch and are calculated according to

$$D_i = \sum_{j \in N_{d,i}} P_j - \sum_{j \in B_{n(i)}} P_{loss,j} \forall i \in B$$ (7)

$$E_i = \sum_{j \in N_{d,i}} Q_j - \sum_{j \in B_{n(i)}} Q_{loss,j} \forall i \in B.$$ (8)

$N_{d,i}$ is the set of nodes located downstream of the $i$-th branch, while $P_j$ and $Q_j$ denote the active and reactive power injections of the $j$-th node, respectively. $B_{n(i)}$ is the set of branches located right after the node $n(i)$. Additionally, $n(i)$ stands for the downstream node connected to the $i$-th branch. Furthermore, (9) and (10) calculate the active ($P_{loss,i}$) and reactive ($Q_{loss,i}$) power losses of the $i$-th branch, respectively.

$$P_{loss,i} = R_i(D_i^2 + E_i^2)/V_{n(i)}^2 \forall i \in B$$ (9)

$$Q_{loss,i} = X_i(D_i^2 + E_i^2)/V_{n(i)}^2 \forall i \in B.$$ (10)

The OLTC operation is modeled by discretely varying the voltage magnitude ($V_0$) of the slack bus as follows:

$$V_0 = V_{hv}/\{m[1 + \text{tap}(\delta/100)]\}$$ (11)

where $V_{hv}$ is the voltage magnitude of the HV grid, $m$ is the voltage transformation ratio, $\text{tap}$ stands for the tap position of the OLTC, and $\delta$ is the percentage variation of the transformation ratio per tap position change.

To maintain the network voltages within permissible limits and to avoid congestion issues, the inequality constraints of (12) and (13) are introduced. Moreover, the active power generation limits of the DG units are considered in (14).

$$V_{min} \leq V_i \leq V_{max} \forall i \in N$$ (12)

$$I_i \leq I_{max,i} \forall i \in N$$ (13)

$$0 \leq P_{dg,i} \leq P_{max,i} \forall i \in N_{dg}$$ (14)

Here, $V_{min}$ and $V_{max}$ are the minimum and the maximum permissible voltage limits determined by the DSO, while $I_i$ and $I_{max,i}$ are the current magnitude and the thermal limit of the $i$-th branch, respectively. Although the Standard EN 50160 poses a maximum voltage variation of $\pm 10\%$ of the nominal voltage [18], many DSOs adopt stricter limits in MV networks. In this paper, the permissible voltage variation is considered equal to $\pm 5\%$. $P_{dg,i}$ and $P_{max,i}$ are the actual and the maximum generation limit of DG unit connected to the $i$-th node. Additionally, (15)-(16) represent the boundary limits of the control variables:

$$Q_{min,i} \leq Q_i \leq Q_{max,i} \forall i \in N_{dg}$$ (15)

$$\text{tap} \in D.$$ (16)

where $N_{dg}$ is the set of network nodes in which the DG units are connected. $Q_i$ is the reactive power produced by either the DG unit connected to the $i$-th node, whereas $Q_{min,i}$ and $Q_{max,i}$ are the corresponding permissible limits. Finally, $D$ is the discrete set of the available tap positions.

IV. System Under Study

The long-term performance of the above-mentioned control schemes is evaluated by performing a probabilistic analysis on the radial 20 kV network of Fig. 3. This network consists of 45 nodes, while the length and the impedance of the lines are presented in Tables I and II, respectively. The DG units and the loads are numbered according to their connection node, while their rated active power is shown in Table III. For reasons
of simplicity, only one type of DG is considered, i.e., PV units. Loads operate with constant power factor equal to 0.95 inductive, while the minimum power factor of the PV units, considering nominal generation conditions, is equal to 0.85. Finally, a 150 kV/20 kV, 50 MVA transformer is considered with a short-circuit voltage 12% and full load losses equal to 0.5%. The OLTC range (\(D\)) is \(\pm 8\) with a voltage variation (\(\delta\)) of 1.67% per tap change.

To implement the probabilistic analysis, Monte Carlo-based time-series simulations are performed for one day assuming the typical generation and consumption profiles of Fig. 4 with one minute time interval. These profiles are arbitrary allocated among the PV units and the loads of the network. In each time instant, a 500 sample of input data are randomly generated to model load and generation uncertainties. It is assumed that PV units uncertainty follows a beta distribution, where coefficients \(\alpha\) and \(\beta\) are equal to 2.06 and 2.50, respectively, and are obtained using original historical data [20]. Additionally, the arbitrary generated values of loads follow a normal distribution with mean values equal to the corresponding deterministic values of Fig. 4, whereas the standard deviation equals to 5% of the mean values [21].

V. SIMULATION RESULTS

In this paper, the performance of one centralized and three decentralized voltage control strategies is thoroughly evaluated. The centralized method is the mixed-integer nonlinear optimal power flow (OPF) problem of (3)-(16), which is solved in GAMS using the BONMIN solver [22]. The three decentralized control schemes include the \(Q(V)\) droop characteristic and two different implementations of the \(\cos \phi (P)\) droop characteristic, namely the \(\cos \phi (P)\)–A and \(\cos \phi (P)\)–B.

The former implements the droop characteristic as presented in Fig. 1, while the latter considers only the negative part of Fig. 1, where PV units absorb reactive power. The corresponding settings of the droop characteristics are presented in Table IV. In the decentralized methods, the automatic voltage regulation (AVR) method is assumed for the OLTC, with a target voltage at the MV busbar equal to 1.05 p.u., a 4-minute time delay, and deadband equal to 0.018 p.u.

TABLE III
RATED ACTIVE POWER OF PV UNITS AND LOADS

<table>
<thead>
<tr>
<th>Nodes</th>
<th>MW</th>
<th>Nodes</th>
<th>MW</th>
<th>Nodes</th>
<th>MW</th>
<th>Nodes</th>
<th>MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.60</td>
<td>6.14</td>
<td>2.00</td>
<td>10,20,25</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.70</td>
<td>8.37</td>
<td>-0.20</td>
<td>4,22,27,38,40</td>
<td>-0.60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-0.90</td>
<td>15.24</td>
<td>1.50</td>
<td>5,17,26,30,41</td>
<td>-0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>0.50</td>
<td>29.35</td>
<td>-0.80</td>
<td>9,19,28,42,44</td>
<td>-0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>-1.00</td>
<td>32.33</td>
<td>-0.70</td>
<td>2,11,21,34,36,43</td>
<td>-0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>-1.20</td>
<td>3.16,18</td>
<td>-0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE IV
SETTINGS OF THE DROOP CHARACTERISTICS (P.U.)

<table>
<thead>
<tr>
<th>(p_1)</th>
<th>(p_2)</th>
<th>(p_2)</th>
<th>(V_1)</th>
<th>(V_2)</th>
<th>(V_3)</th>
<th>(V_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\cos \phi (P))–A</td>
<td>0.12</td>
<td>0.5</td>
<td>0.88</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(\cos \phi (P))–B</td>
<td>–</td>
<td>0.5</td>
<td>0.88</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>(Q(V))</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.95</td>
<td>0.965</td>
<td>1.035</td>
</tr>
</tbody>
</table>

Indicative results of the Monte Carlo-based time-series simulations are presented in Figs. 5-10 for the OPF and the decentralized voltage control strategies. In particular, the occurrence of the overall reactive power consumption of PV units is presented in Fig. 5, while the occurrences of the maximum and minimum network voltages are depicted in Fig. 6 and Fig. 7, respectively. Furthermore, results of the probabilistic analysis, concerning the tap position of the OLTC and the daily number of tap changes are presented in Fig. 8 and Fig. 9. Finally, the occurrence of the daily energy losses is presented in Fig. 10.

According to Fig. 5, it can be observed that the decentralized \(Q(V)\) droop characteristic results in excessive reactive power consumption during high generation periods, which is mainly caused by two reasons. The first one is related with the inherent problem of the droop characteristic to unnecessarily absorb reactive power in voltages below the maximum permissible limits, which can be also verified by Fig. 2 and Table IV. The second reason is related with the AVR operation of the OLTC, which reduces the tap position during high generation periods to maintain the MV busbar voltage at 1.05 p.u. This results in increased network voltages, which in turn forces the activation of the \(Q(V)\) droop characteristic.

Considering the \(\cos \phi (P)\)–B method, overvoltages cannot be fully addressed as shown in Fig. 6. Furthermore, in the \(\cos \phi (P)\)–A method, there exist time instants where PV units
provide reactive power to the grid, having an adverse effect on the mitigation of network overvoltages. This also increases the number of daily tap changes, as presented in Fig. 9. This is an inherent drawback of the \( \cos \phi(P) \) methods, since the reactive power is calculated based on the power factor, neglecting the reactive power capability of the DG units. As a result, the reactive power potential of the PV units remains unexploited. Furthermore, the power factor depends only on the active power injection, ignoring the current state of the network operation.

On the other hand, the \( Q(V) \) droop characteristic method mitigates overvoltages to a greater extent compared to the other decentralized methods. However, voltage violations still persist due to the uncoordinated operation of the PV units. Considering undervoltages, all methods ensure that voltages are above the lower permissible limits, which is also observed in Fig. 7.

All the examined methods except the decentralized \( \cos \phi(P) \)–B implementation lead to similar number of daily tap changes. Generally, the OPF method presents a reduced number of daily tap changes against the decentralized \( Q(V) \) method and similar to the \( \cos \phi(P) \)–A implementation. On the other hand, the \( \cos \phi(P) \)–A implementation presents the smallest daily energy losses without, however, ensuring the network operation within permissible voltage limits, as shown in Fig. 6. Moreover, the \( Q(V) \) method presents increased and more widely distributed energy losses which is caused due to the droop characteristic and the AVR operation of the OLTC. The OPF method ensures the safe and reliable network operation, while also minimizing network losses. Nevertheless, the main drawback of this type of voltage control strategy is that it presents an increased computational complexity and possible suboptimal solutions.
According to the probabilistic analysis, it can derived that the examined methods present drawbacks, concerning the overvoltage mitigation, reactive power absorption and computational complexity. Therefore, there is a strong need that new methods should be developed to effectively address these issues and can be readily incorporated into the grid codes for the DG interconnection.

VI. CONCLUSIONS

In this paper, a probabilistic analysis is performed to thoroughly evaluate the long-term performance of the most well-established voltage control strategies. This is attained by conducting Monte Carlo-based time-series simulations on an extended radial MV networks, assuming generation and loads follow specific distribution patterns. The simulation results reveal drawbacks in all the examined control strategies, including also the centralized and decentralized control schemes. Therefore, there is a strong need to develop new voltage control methods to address these drawbacks.

REFERENCES


