Power Flow Analysis of Islanded AC Microgrids

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Abstract—This paper presents a methodology for the steady-state analysis, i.e., power flow analysis, of islanded AC microgrids (MGs), operated under the droop control scheme. In the proposed framework, the power flow is computed using conventional power flow methods. For this purpose, a slack bus is considered to be connected to a random node of the examined MG. Subsequently, an iterative procedure is applied to determine the operational conditions, i.e., voltage level, frequency, active and reactive power absorption/injection, of this slack bus as well as of the rest MG nodes. The objective of the proposed iterative procedure is to force active and reactive power, flowing through the slack bus, to zero, emulating this way the islanded operation of the MG. To evaluate the performance of the proposed methodology, time-domain simulations are conducted on a low-voltage MG, using detailed models for all system components. The corresponding results are compared with those derived using the proposed methodology. In all cases, trivial differences are observed, validating the accuracy and the robustness of the proposed approach.

Index Terms—Droop control method, dynamic analysis, islanded microgrids, power flow analysis.

I. INTRODUCTION

To facilitate the dynamic and steady-state analysis of MGs, several mathematical models and techniques have been proposed in the literature [1]. Concerning steady state analysis, power flow tools are essential for the secure and reliable operation of MGs [2], [3]. These tools can be used for planning purposes, network optimization, reconfiguration studies, security analysis as well as for the initialization of system dynamic analysis [3].

The power flow problem can be easily tackled in grid-connected MGs by applying conventional power flow (CPF) methods, such as the Newton Raphson and the Gauss-Seidel [4]. These methods are based on the concept of the slack bus, which is used to simulate the utility grid [4]. In particular, the slack bus imitates the behavior of a large synchronous machine, providing/absorbing the deficit/surplus of active and/or reactive power. In this concept, the grid frequency is predetermined, while voltage magnitude of the slack bus is assumed constant and a priori known. Distributed generation (DG) units are represented as PV or PQ buses [5], [6], i.e., as buses with pre-specified active power and voltage magnitude or buses with pre-specified active and reactive power.

However, CPF methods cannot be used for the analysis of droop-controlled islanded MGs [7], [8]. The modeling of these systems differs from grid-connected MGs in the following aspects: a) In these systems, the concept of a slack bus operating at fixed frequency and voltage is not valid [2], since node voltages and network frequency are determined by the operating conditions of the MG loads and DG units and they are controlled by the corresponding droop characteristics [7], [8]. b) Network reactances cannot be considered constant, since they are considerably affected by the frequency changes.

A potential solution involves the detailed modeling of the examined MG in standard time-domain (TD) simulation software, such as PSCAD, Simulink, PSIM, among others, and the simulation of the MG behavior until the transient phase is damped out [9]. This approach provides very accurate and reliable results. However, the calculation of the steady-state solution using TD simulations generally requires high execution times [9]. Hence, the applicability of this approach for the analysis of large-scale MGs is highly questionable.

To avoid time-consuming TD simulations and to facilitate the power flow analysis of AC islanded MGs, several approaches have been proposed in the literature [2], [8]–[16]. More specifically, a particle swarm optimization method is proposed in [2]. However, as discussed in [10], the use of evolutionary methods for power flow calculation may lead to considerably high execution times as well as to inconsistent results due to initialization failures. In [8] and [11], modified Newton Raphson methods are implemented. These methods are accurate but they are characterized by increased computational burden, since they require the inverse of the Jacobian Matrix. In [12], a Newton trust-region method is proposed for the power flow analysis of balanced and unbalanced islanded MGs. The method is based on nonlinear equations, used to simulate the operating modes of DG units. The method is accurate but computationally intensive for extended MGs. In [13]–[15], modified backward-forward sweep load flow methods for islanded MGs are proposed. These methods are fast and accurate but they can only be applied for the analysis of radial or weakly-meshed distribution grids [17]. In our
previous work [16], a novel methodology, based on a two-nested-loop iterative procedure, was developed to extend the applicability of CPF methods for droop-based islanded MGs. The frequency of the MG and active power injections of the DG units are computed in the first loop. Afterwards, the second loop is used to calculate node voltages and reactive power injections of the DG units. However, modeling of voltage dependent droop curves using iterative procedures, may result in numerical oscillations of the injected power and network voltages. As discussed in [18] and [19], in extended networks with high penetration of DG units, these spurious oscillations may lead to convergence issues. Therefore, in these cases, the method of [16] may fail to provide consistent results.

To overcome this issue, in this paper, the method of [16] is further extended to enhance the power flow analysis of large-scale islanded MGs. Towards this objective, a third loop is added in the formulation of [16]. The aim of this loop is to simulate with higher accuracy the $Q - V$ droop control. The performance of the proposed methodology is evaluated on a low-voltage (LV) MG by means of TD simulations conducted using the PSIM software. The examined LV MG constitutes a modified version of the European LV residential distribution grid proposed by Cigre Task Force C6.04 [20]. Grid and converter models are modelled with high details, aiming to develop a benchmark LV MG suitable for the testing and analysis of power flow methodologies oriented to the analysis of islanded MGs.

II. DROOP-BASED ISLANDED MGs

The power sharing of active and reactive power among the DG units of islanded MGs can be achieved using the $P - f$ and $Q - V$ droop curves, respectively [21]. These droop curves, can be mathematically expressed using the following set of equations:

\[
f_{\text{grid}} = f_{\text{max}} - m_i P_i \quad \forall i \in N_{\text{dg}} \tag{1}
\]

\[
V_i = V_{\text{max}} - n_i (Q_i + Q_{\text{max},i}) \quad \forall i \in N_{\text{dg}} \tag{2}
\]

where

\[
m_i = \frac{f_{\text{max}} - f_{\text{min}}}{P_{\text{max},i}} \quad \forall i \in N_{\text{dg}} \tag{3}
\]

\[
n_i = \frac{V_{\text{max}} - V_{\text{min}}}{2 Q_{\text{max},i}} \quad \forall i \in N_{\text{dg}}. \tag{4}
\]

Here, $N_{\text{dg}}$ denotes the number of the installed DG units. $V_{\text{max}}, V_{\text{min}}, f_{\text{max}}$ and $f_{\text{min}}$ are the maximum and minimum operational limits of voltage and frequency, respectively. In this paper, these limits are defined according to Standard EN50160 [22]. Additionally, $Q_{\text{max},i}$ is the maximum available reactive power of the $i$-th DG unit, while $P_i$ and $Q_i$ are the injected active and reactive power of the $i$-th unit, respectively. $m_i$ and $n_i$ are the corresponding droop coefficients [21].

III. PROPOSED METHOD

As discussed in the introduction, CPF methods cannot be used for the power flow analysis of islanded MGs, since they require the existence of a slack bus, operating at constant and pre-determined voltage and frequency. Therefore, to apply these methods for the power flow analysis of islanded MGs, the operational conditions of the slack bus must be available prior to the power flow solution [16]. To address this issue, in this paper, the three-nested-loop iterative procedure of Fig. 1 is proposed. The objective of this iterative procedure is to properly adjust the network frequency ($f_{\text{grid}}$) and the voltage magnitude of the slack bus ($V_{sb}$) in order to force active and reactive power flowing through the slack bus to zero, emulating this way the islanded operation mode. The proposed iterative method consists of the following steps:

**Step-1:** The initial conditions are applied. In particular, $V_{sb}$ and $f_{\text{grid}}$ receive the maximum permissible values, i.e. $V_{\text{max}}$ and $f_{\text{max}}$, respectively. Additionally, variables $V_{sb}$ and $V_{sb}$ are set equal to $V_{\text{max}}$ and $V_{\text{min}}$, respectively. A further insight on these variables will be given in Step-13. For the initialization, the active power injections of all DG units are considered equal to zero, while reactive power injections are considered equal to $-Q_{\text{max},i}$. In each power flow calculation, active and reactive power injections are stored in two discrete vectors denoted for the rest of the paper as $P_{\text{dg}}$ and $Q_{\text{dg}}$, respectively. Additionally, at this step, the user defines the value of the integral term $k$, used to calculate reactive power injections of DG units. A further insight on this integral term is provided in Step-13. Finally, at this step, the user defines the maximum number of iterations, i.e., $\text{MaxIter}$, and the tolerances for the convergence of the three loops, i.e. $\text{tolp}$, $\text{tolv}$, $\text{tolq}$.

**Step-2:** Line and load impedances are computed at $f_{\text{grid}}$.

**Step-3:** Subsequently, a power flow analysis is performed and the value of counter $c$ is set to 1. This counter is used to monitor the number of iterations performed during the second loop of the proposed algorithm. Further information concerning this counter can be found in Steps 6 to 11.

**Step-4:** The active power flowing through the slack bus ($P_{sb}$) as well as the network voltages ($V'$) are computed.

**Step-5:** A convergence check is performed to evaluate if the absolute value of $P_{sb}$, i.e., $|P_{sb}|$, is greater than the predefined tolerance $\text{tolp}$. If so, $P_{sb}$ must be reallocated to all installed DG units in order to zero the active power flowing through the slack bus. To achieve this objective, all DG units must adjust their injected active power by a $\Delta P_i$ amount. This amount can be mathematically expressed using (5) and (6). Please note that (6) is imposed due to the use of the droop control mechanism [10]. Then, vector $P_{\text{dg}}$ is updated by adding the corresponding $\Delta P_i$ amounts. Subsequently, the new $f_{\text{grid}}$ is calculated using (1) and the procedure moves back to Step 2. Then, a new power flow is performed using the updated values of $P_{\text{dg}}$ and $f_{\text{grid}}$. When $|P_{sb}|$ becomes lower than $\text{tolp}$, the algorithm moves to Step-6.

**Step-6:** The value of counter $c$ is increased by one.
Step-1: Define $tol_P$, $tol_Q$, $tol_V$, MaxIter, $k$
Step-2: Build network with $V_b$ and $f_{grid}$
Step-3: Perform power flow analysis
Step-4: Calculate $P_{slc}$ and $V^c$
Step-5: if $|P_{slc}| \leq tol_P$ then
  Update $f_{grid}$ and $P_0$ using (1), (5) & (6)
else
  $c \leftarrow c + 1$
Step-6: if $c \leq MaxIter$ then
  $c \leftarrow c + 1$
else
  End
Step-7: Use (2) & $V^{c-1}$ to calculate $Q_{sb}$
Step-8: Use (7) to calculate $Q_{sb}$
Step-9: Perform power flow analysis
Step-10: if $|V^c - V^{c-1}| \leq tol_V$ then
  End
else
  Calculate $Q_{sb}$
Step-11: if $|Q_{sb}| \leq tol_Q$ then
  Update $V_{sb}$ (see Algorithm 1)
else
  End
Step-12: A comparison of all node voltages between the last two iterations is performed. If $tol_V$ criterion is satisfied, the algorithm proceeds to Step-12, otherwise the algorithm moves back to Step-6 and new reactive power injections are computed.
Step-13: Using the results of the last power flow, the reactive power flowing through the slack bus, i.e., $Q_{sb}$, is calculated.
Step-14: If the absolute value of $Q_{sb}$, i.e., $|Q_{sb}|$, is less than $tol_Q$, the algorithm moves to Step-14 and terminates. Otherwise, a new $V_{sb}$ must be defined to zero the reactive power flowing through the slack bus. More specifically, a negative $Q_{sb}$ value indicates a surplus of reactive power in the MG, absorbed from the slack bus. Therefore, according to (2), $V_{sb}$ must be increased to force DG units to reduce their reactive power injection. On the contrary, if $Q_{sb}$ is positive, $V_{sb}$ must be reduced. The new value of $V_{sb}$ is determined through a binary search method [16]. The binary search algorithm is presented in Algorithm 1. In Algorithm 1, $V_{sb}$ and $V_{lb}$ are the upper and lower limits of the searching interval, while $p$ and $n$ are the corresponding previous and new values. Once the new $V_{sb}$ is defined, the procedure moves back to Step-2.
Step-15: The procedure terminates providing the power flow solution of the examined MG.

### Algorithm 1 Binary search for $V_{sb}$

1: if $Q_{sb} > 0$ then
2:   set $V_{sb}^{n}$ to $V_{sb}^{k} - (V_{sb}^{p} - V_{sb}^{l})/2$, $V_{lb}$ to $V_{lb}^{p}$, $V_{ub}$ to $V_{ub}^{k}$
3: else
4:   set $V_{sb}$ to $V_{sb}^{n} + (V_{sb}^{p} - V_{sb}^{l})/2$, $V_{lb}$ to $V_{lb}^{p}$, $V_{ub}$ to $V_{ub}^{k}$
5: end if

The value of $k$ can vary from small to large values. Small values result in small execution times, but cannot guarantee the convergence. On the other hand, large values exhibit slow convergence rate, but result in more robust and accurate results [18], [19].

**Step-10:** Voltages of all MG nodes, i.e., $V^{c}$, are computed through a power flow analysis using $P_{dg}$, $Q_{c}^c$, and $f_{grid}$.

**Step-11:** A comparison of all node voltages between the last two iterations is performed. If $tol_V$ criterion is satisfied, the algorithm proceeds to Step-12, otherwise the algorithm moves back to Step-6 and new reactive power injections are computed.

**Step-12:** Using the results of the last power flow, the reactive power flowing through the slack bus, i.e., $Q_{sb}$, is calculated.

**Step-13:** If the absolute value of $Q_{sb}$, i.e., $|Q_{sb}|$, is less than $tol_Q$, the algorithm moves to Step-14 and terminates. Otherwise, a new $V_{sb}$ must be defined to zero the reactive power flowing through the slack bus. More specifically, a negative $Q_{sb}$ value indicates a surplus of reactive power in the MG, absorbed from the slack bus. Therefore, according to (2), $V_{sb}$ must be increased to force DG units to reduce their reactive power injection. On the contrary, if $Q_{sb}$ is positive, $V_{sb}$ must be reduced. The new value of $V_{sb}$ is determined through a binary search method [16]. The binary search algorithm is presented in Algorithm 1. In Algorithm 1, $V_{sb}$ and $V_{lb}$ are the upper and lower limits of the searching interval, while $p$ and $n$ are the corresponding previous and new values. Once the new $V_{sb}$ is defined, the procedure moves back to Step-2.

**Step-14:** The procedure terminates providing the power flow solution of the examined MG.

### IV. System Under Study

The system under study is presented in Fig. 2 and consists of a 0.4 kV islanded MG. This system is based on the topology of the European LV residential distribution grid [20]. The notation and the numbering of nodes in Fig. 2 is according to [20]. To derive the LV MG, the following modifications have been applied on the benchmark model of [20]. First, six (6) DG units are connected to nodes R2, R5, R7, R10, R12, and R15. The droop coefficients of all DG units are presented...
in Table I. Additionally, the load connected in node R1 of the benchmark system is intentionally omitted from the LV MG of Fig. 2. As discussed in [20], this load represents additional feeders served by dedicated transformers, i.e. this load does not constitute a part of the examined feeder. Thereby it is omitted from the analysis. All other network loads as well as network lines are modeled as discussed in [20].

Details concerning the modeling of the converters, used for the interconnection of the DG units, are provided in Figs. 3 to 5. In particular, the topology of the employed converter is presented in Fig. 3, where the power circuit is depicted with black lines and the control circuit with red lines. $V_a^s$, $V_b^s$, and $V_c^s$ denote the voltage control signals in the abc framework, while $V_a$, $V_b$, and $V_c$ are the corresponding grid voltages. $I_a^{bf}$, $I_b^{bf}$, and $I_c^{bf}$ are the currents before the LC filter, while $I_d^{bf}$, $I_q^{bf}$, and $I_0^{bf}$ denote the current injections at the grid in the abc framework. The inductance ($L$) and the capacitor ($C$) of all LC filters are equal to 2 mH and 15.5 μF, respectively. The transition frequency of the switches for all DG converters is set to 9950 Hz. The injected active and reactive powers are computed as presented in Fig. 4, where $V_d$, $V_q$, $I_d^{bf}$ and $I_q^{bf}$ denote grid voltages and current injections in the $dq$ framework, respectively. Abbreviation LPF denotes the existence of a low pass filter. The cutoff frequency for all LPFs is equal to 10 Hz. Control signals are computed based on the approach presented in Fig. 5. Here, $I_d^{bf}$ and $I_q^{bf}$ denote the total current injection in the $dq$ system, while $V_d^s$ and $V_q^s$ the corresponding voltage control signals. $\omega$ is the angular frequency and $\theta$ the corresponding angle between $0dq$ and $abc$ reference frames. $PI_{V1}$, $PI_{C1}$, $PI_{V2}$, and $PI_{C2}$ denote the required PI controllers. The corresponding settings are presented in Table II.

![Fig. 2. Examined LV MG.](image)

![Fig. 3. Converter topology.](image)

![Fig. 4. Active and reactive power calculation.](image)

![Fig. 5. Control process.](image)

**TABLE I**

DROOP COEFFICIENTS $m$ (HZ/kW) & $n$ (V/kVAR)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>DG1</th>
<th>DG2</th>
<th>DG3</th>
<th>DG4</th>
<th>DG5</th>
<th>DG6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>0.050</td>
<td>0.066</td>
<td>0.050</td>
<td>0.040</td>
<td>0.040</td>
<td>0.066</td>
</tr>
<tr>
<td>$n_1$</td>
<td>1.613</td>
<td>2.151</td>
<td>1.613</td>
<td>1.291</td>
<td>1.291</td>
<td>2.151</td>
</tr>
</tbody>
</table>

**TABLE II**

SETTINGS OF PI CONTROLLERS

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$PI_{V1}$</th>
<th>$PI_{V2}$</th>
<th>$PI_{C1}$</th>
<th>$PI_{C2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>0.45</td>
<td>0.45</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_i$</td>
<td>4.5</td>
<td>4.5</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
V. NUMERICAL VALIDATION

A. Time-domain Simulations

Initially the LV MG of Fig. 2 is implemented in full detail in the PSIM software and the power flow problem is solved by performing TD simulations. The developed models can be found in [23]. Dynamic responses of active and reactive power injections of all DG units as well as the dynamic response of MG frequency are presented in Figs. 6a to 6c, respectively. Based on these results, it can be perceived that the steady-state of the MG is established approximately at around \( t=3 \) s. However, to ensure that all dynamic phenomena have been properly damped, the TD simulation continues up to \( t=25 \) s. To simulate the first three seconds and to reach the steady-state condition, PSIM requires approximately 2958 s. All simulations were conducted using an Intel Core i7-930, 2.8 GHz, RAM 24 GB, personal computer.

The MG frequency in the new steady-state is equal to 49.36869 Hz. Active and reactive power injections of all DG units are reported in Table III, while the voltage profile along the MG is presented in Fig. 7.

B. Simulation Results

In this subsection the proposed method is implemented using the MATLAB and the OpenDSS software. The former is used to implement the flowchart of Fig. 1, while the latter is utilized as a phase-domain power flow solver [18]. \( tol_p, tol_q \) and \( tol_v \) are set to 0.1 W, 0.1 V Ar, and 0.00001 V, respectively. \( MaxIter \) and \( k \) are assumed equal to 100 and 2, respectively.

Initially, the slack bus is randomly placed at node R1. The total execution time of the proposed method is 11.3 s. The MG frequency, as calculated by the proposed method, is equal to 49.36874 Hz. Compared to detailed TD simulations an absolute percentage error equal to 0.00012 % is observed. Active and reactive power injections of all DG units are listed in Table III. As shown, a near-zero mismatch, compared to TD simulations, is observed. Additionally, the voltage profile along the MG is depicted in Fig. 7. Once again, only trivial differences compared to TD simulations are recorded, validating the accuracy of the proposed method. Indeed, the maximum absolute percentage error between the TD simulation and the proposed method is only 0.026 % and observed for node R1.

The proposed method yields almost identical results regardless the location of the slack bus. To further support this argument, the power flow solution is derived assuming different locations for the slack bus. More specifically, eighteen distinct test cases are considered. In each test case, the slack bus is assumed to be connected at a different MG node. The mean execution time of the eighteen power flows is 11.19 s, validating the low computational burden of the proposed method. For each test case, the absolute percentage errors, concerning the voltage magnitude of all MG nodes, between the proposed method and the TD simulation, are computed. The corresponding results are analyzed in the boxplots of
trivial differences are observed.

Fig. 8. In all cases, regardless the location of the slack bus, assuming different locations for the slack bus.

VI. CONCLUSIONS

In this paper, a new method is proposed to accurately solve the power flow problem of islanded AC MGs. Towards this objective, a three-nested-loop iterative procedure is developed to extend the applicability of CPF methods for the analysis of islanded MGs. The performance of the proposed method is validated on an eighteen-bus LV MG using TD simulations. In all test cases, the method results in very accurate and robust estimates, presenting also low computational burden. Therefore, it can be concluded that the proposed method constitutes a reliable tool which can be used for the power flow analysis of islanded MGs.

Future work will be conducted to extend the applicability of the developed method for the analysis of unbalanced MGs as well as for the analysis of islanded MGs operated with hierarchical control. Finally, further steps aim to include the virtual impedance concept in the proposed formulation.

REFERENCES