Harmonic Mitigation in Low-Voltage Distribution Networks: Sensitivity Analysis

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***Abstract*—The increased penetration of converter-interfaced distributed renewable energy sources (CI-DRES) along with the existence of non-linear loads (NLLs) might cause severe power quality problems in distribution networks (DNs). Currently, various methods have been proposed so that voltage source converters (VSCs) of the CI-DRES act also as harmonic filters, however, most of them mitigate the problem locally without examining which CI-DRES within the DN would be most suitable to provide such service. In this paper the mathematical framework and a sensitivity analysis is performed in order to evaluate the contribution of each CI-DRES in the decentralized harmonic filtering process, with the presence of NLLs within a low-voltage (LV) DN based on the CI-DRES location and available capacity. The available capacity is an important parameter, since in the future, the CI-DRES might provide simultaneously several services, e.g. reactive power or virtual inertia. The mathematical framework for the quantification of the CI-DRES contribution is validated via simulations in the time domain and via harmonic power flows in the benchmark CIGRE LV network under various mixtures of CI-DRES/NLLs. The derived conclusions will pave the path, so that the active filtering can be treated as a new ancillary service to be provided by CI-DRES and introduced in respective markets.**

***Index Terms*—Active filters, Ancillary Service, Distributed Generation, Harmonic Mitigation**

I. Introduction

The high proliferation of converter interfaced distributed renewable energy sources (CI-DRES) has caused several problems into the distribution networks (DNs), e.g. reverse power flows, voltage regulation issues, power quality problems, [1]–[3]. In addition, the conventional electromechanical loads are being replaced by power-electronic devices, e.g., led lighting, variable-speed drives, etc., which distort the current waveform with low- and high-frequency harmonics. For this reason, they have negative impacts on the DNs equipment and loads, [1]–[3], e.g., overheating, overload of the neutral wires, electromagnetic interferences, etc. Therefore, international regulations require that the harmonic content of the DNs is maintained below specific limits [2]. As examples, standards IEEE Std 519, [2], and IEC 61000-3-6, [4], define limits for individual and total harmonic distortion (HD) of the CI-DRES voltage and current at its point of common coupling (PCC) with the grid. On the other hand, the advanced controllability of the voltage source converters (VSCs) that interface the CI-DRES with the DN, can be exploited to reduce the harmonic distortion. Hence, the CI-DRES may inject controlled harmonic components in addition to the active power with the aim to improve the power quality in the DN. Up to now, the research studies with respect to the presence of harmonics and their mitigation can be divided into the following categories, [5]:

(i) the impact of the CI-DRES penetration and NLLs on the DNs, which pose a limit to the DRES Hosting Capacity (HC) and the allowable CI-DRES penetration considering the HD limits: In [6], the impact of the CI-DRES generated HD components on the HC of DNs is investigated taking into account the limits imposed by the IEEE 519 Std. In [7] the importance of including HD limits in the HC assessment is highlighted. In [8] the Harmonic-Constrained HC term is introduced to stress out that the power system HC can be determined by considering only the voltage HD limits.

(ii) the optimal installation and size of active and passive filters considering the limits on the DN HC, [9]–[11]: single-tuned passive harmonic filters have been designed to be placed in an industrial system with NLLs and targeting to minimize the average voltage total HD (THD_V) of the DN based on the fundamental-frequency power flow and on Decoupled Harmonic Power Flows (DHPFs). The optimal filter sizing is solved as mixed integer optimization problem considering total and individual HD percentages.

(iii) active filtering (AF) control techniques and the operation of CI-DRES as active power filters (APFs): These methods assume that the voltage HD is a phenomenon that should be mitigated locally (decentralized approaches). In [12] it is suggested that CI-DRES could provide harmonic mitigation as an ancillary service (AS) without additional purchase costs if its apparent power is sufficient. In this way, the installation of large-scale central APFs could be avoided, since they are an expensive solution. The CI-DRES AF control approaches are classified into two main categories, [12]:

- virtual impedance-based methods: a virtual impedance (resistance, [13], [14] or conductance, [5]) can emulate the effect of physical impedance without the need to connect a real component to a system.
- active harmonic filtering-based methods, where the main objective is to produce an appropriate compensating current in the VSC. This current is generated by comparing the load current with a predetermined reference current.

The use of a central active or passive filter at system level in [9], [10], [13], [15] exhibits the following drawbacks, [9]: (a) possible parallel resonance of the filter with other electrical components of the DNs; (b) after their sizing the voltage and current of the filter elements should be checked to avoid any detrimental increase in their rating compared to their

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fundamental rating. In a possible failure of the filter, the AF could lead to the "whack-a-mole" effect, [13], [16], where the mitigation of one harmonic frequency voltage (HFV) could lead to the increase of another HFV. In order to avoid the whack-a-mole effect, the HD in multi-bus electrical grids may be mitigated by connecting an APF to each grid node, [15].

There yet exist several other gaps for addressing the HD within DNs. Recently issued Standards, e.g. [17], [18], have provided specifications for the operation of CI-DRES under the exchange of reactive power (RP) for voltage regulation. The untapped capacity of the CI-DRES can be exploited to provide harmonic mitigation as an AS that could be tradable in specific AS markets. Nevertheless, the harmonic mitigation as an AS to be offered by the CI-DRES needs to be co-evaluated along with other AS keeping in mind that the CI-DRES may exceed its thermal capacity. Until now, most of the AF methods rely on local controllers without considering the effects in the rest of the DN and without examining which CI-DRES within the DN would be most suitable to provide such service. Hence, the AF operation by CI-DRES has been treated in a decentralized manner, i.e., the CI-DRES close to a NLL should compensate the low-order harmonic emissions of the NLL. This action is similar to decentralized voltage regulation approaches. The sensitivity of a CI-DRES location with respect to the NLL low-order harmonic frequencies (HFs) at DN level has not been studied yet in the technical literature. Consequently, when performing a decentralized harmonic mitigation control, the contribution of a CI-DRES needs to be quantified considering the CI-DRES location (with respect to the NLL or within the DN) and in terms of its thermal capacity.

This paper attempts to fill the identified gaps as follows: A mathematical framework is formulated towards the evaluation of the sensitivity of the CI-DRES location within a LV DN under the presence of NLLs. This framework provides a further insight on the quantification of the contribution of each CI-DRES in the AF process considering the CI-DRES thermal capacity as a parameter. As it is evident in [6], [19], [20] the most prevailing voltage harmonics within DNs are the low-order ones, i.e., $5^{th}, 7^{th}, 11^{th}, 13^{th}$. In order to respect the CI-DRES thermal limit and not overload the CI-DRES with higher order HFs, in [5] and [14] decentralized virtual conductance and resistance approaches respectively have been proposed to mitigate simultaneously the above HFs within DNs, thus, avoiding the "whack-a-mole" effect. In this paper, the sensitivity analysis is performed considering these HFs in order to quantify the contribution of each CI-DRES to the decentralized APF process. The results prove that there exist CI-DRES which are more critical during the APF process, hence, they may exhaust their capacity compared to others within the same LV DN. This evaluation will form a basis for the development of a new distributed harmonic mitigation algorithm is developed based on the sensitivity matrix at each harmonic. This procedure has been already implemented in state-of-the-art voltage regulation methods, which employ the CI-DRES available RP, [21]. Furthermore, if the contribution of each CI-DRES is quantified, the operational costs could be evaluated based on detailed cost-functions [22] and this AS could be remunerated in future respective markets.

The rest of the manuscript is organised as follows: Section II firstly analyses the technical background and then, presents the theoretical background and mathematical framework for the sensitivity analysis. In Section III simulation results are presented to demonstrate the validity of the mathematical framework. Finally, Section IV closes the paper with its main findings and proposes new directions for further research.

II. Sensitivity Analysis

In this section, the mathematical formulation for the sensitivity analysis is described based on specific technical background reported in the state-of-the-art.

A. Proposed DN Modelling

This section presents the DN model which is used in the DHPFs and the power flow at the fundamental frequency f. The current flowing through the branch that connects node i with j is calculated as:

$$\bar{I}_{ij}^x = \bar{Y}_{ij}^x \bar{V}_{ij}^x \quad \forall x \in \{f, h\} \tag{1}$$

where $\bar{V}_{ij}^x$ and $\bar{I}_{ij}^x$ denote the complex voltage drop and the complex branch current between nodes i and j, respectively. The superscript x denotes the examined frequency that can be either f or a higher HF (h). The term $\bar{Y}_{ij}^x$ stands for the series admittance of the line connecting node i with node j at the examined frequency x. The corresponding complex current at node i ($\bar{I}_i^x$) can be generally expressed as follows:

$$\bar{I}_i^x = \sum_{k \in N \setminus \{i\}} \bar{Y}_{ki}^x \bar{V}_k^x - \left(\sum_{k \in N \setminus \{i\}} \bar{Y}_{ki}^x \right) \bar{V}_i^x \tag{2}$$

where N is the set of the network nodes. At fundamental frequency f, (2) is transformed to the well-established power flow equations, [21]:

$$P_i = V_i \sum_{j \in N} V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)], \quad \forall i \in N \tag{3}$$

$$Q_i = V_i \sum_{j \in N} V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)], \quad \forall i \in N \tag{4}$$

where V_i and θ_i denote the magnitude and angle of the voltage node i, respectively. Furthermore, G_{ij} and B_{ij} are the real and imaginary part of the series admittance of the line that connects node i with node j. P_i and Q_i stand for the net injected active and RP at node i that are calculated according to

$$P_i = P_i^G - P_i^L \quad \forall i \in N \tag{5}$$

$$Q_i = Q_i^G - Q_i^L \quad \forall i \in N \tag{6}$$

where P_i^G, Q_i^G, P_i^L, and Q_i^L are the active and RP of the CI-DRES and load connected to node i, respectively. Based on this analysis, it is evident that CI-DRES and loads are modelled as constant PQ sources at f. Eqs. (3)-(6) are employed to model the power flow within the DN at f. The slack bus voltage is set to the rated DN voltage and with a zero angle reference. The shunt capacitances of the lines have been neglected at f due their reduced influence at DN levels.

Regarding the DHFPs, (2) and (7) are employed to model the DN at HFs. For the present analysis, it is assumed that the slack bus does not exhibit any HD_V, hence, at HF h, the voltage angle and magnitude of the slack bus are zero. With respect to the NLL modelling in LV DNs, in [23] four NLL types examined for harmonic analysis of residential loads with OpenDSS in the IEEE LV European network configuration: Norton, Norton with real voltage, Ideal Current Source and Mixed Model (linear load as an admittance and the NLLs as ideal current sources at HFs). It is concluded that when resonances are not present (something that is not expected in LV DNs at low-order HFs), all NLL models bring similar results, while the ideal current source one is less CPU intensive and simulations run faster in OpenDSS. Therefore, in this study, the NLLs are modelled as ideal current sources injecting harmonic currents to the DN.

In case of Medium Voltage (MV) DNs, e.g., [9], [10], shunt capacitors that may exist on the system (e.g., lines or PFC). These shunt capacitors might cause resonances, hence, they should be taken into account in MV DNs. In this case, the current at node i at HF h ($\bar{I}_i^h$) can be calculated as follows:

$$\bar{I}_i^h = -jh\omega C_i V_i^h - \bar{I}_i^{L,h} \tag{7}$$

where ω is the angular frequency, C_i is the shunt capacitance and $\bar{I}_i^{L,h}$ is the *h-th* load harmonic current at node i. In such case, this quantity should be added in (2). In this study, the shunt capacitors of the lines are neglected also in HFs, since the study focuses on LV DNs, where the resistive nature of the lines is prevailing at low-order HFs.

In order to derive the sensitivity matrix, the CI-DRES AF functionality is not activated, hence, at a first stage, the CI-DRES are not present in the DHPFs. At a second step, in order to evaluate the CI-DRES AF functionality, the CI-DRES are modelled as ideal current sources that inject harmonic currents with specific angle to the CI-DRES PCC.

B. Theoretical Background at Fundamental Frequency

Traditionally, the sensitivity matrix is calculated by inverting the Jacobian matrix used in the Newton-Raphson approach for grid power flow analysis, [21], [24]. In [21] the sensitivity analysis method was employed to obtain a quantitative evaluation of the CI-DRES active and RP variation impact on the variation of the voltage magnitude. From power flow analysis, the sensitivity matrix is given by:

$$\mathbf{S}_V = J^{-1} = \begin{bmatrix} \frac{\partial \theta}{\partial \mathbf{P}} & \frac{\partial \theta}{\partial \mathbf{Q}} \\ \frac{\partial |\bar{\mathbf{V}}|}{\partial \mathbf{P}} & \frac{\partial |\bar{\mathbf{V}}|}{\partial \mathbf{Q}} \end{bmatrix} \tag{8}$$

In [25] the term $\frac{\partial |\bar{\mathbf{V}}|}{\partial \mathbf{P}}$ is examined as submatrix, where each element is interpreted as the variation that would happen in the voltage profile at a certain bus i in the case of a hypothetical 1-pu variation in the injection of active power at bus j. In [21] the term $\frac{\partial |\bar{\mathbf{V}}|}{\partial \mathbf{Q}}$ is used to quantify the impact of RP variations on the network voltages. In this paper (8) is used for the sensitivity matrix calculation at f and then, it is adapted for the sensitivity matrix calculation at the HFs.

C. Proposed Theoretical Framework

In order to evaluate the contribution of each CI-DRES in a decentralized control scheme, where the CI-DRES will react on local voltage harmonic measurements at the CI-DRES PCC, the following assumptions are made: (a) The DNO possesses a measuring system for HFVs and HF currents at each low-order HF (**h=5,7,11,13**), especially if it is known that there is a relatively large installation of NLLs (more than 5kVA), e.g., six-pulse rectifiers; (b) Since the DN line impedances and configuration are readily available to the DNO, the DNO can undertake to perform DHFPs and calculate the sensitivity coefficients. This procedure can be repeated e.g., every 1 minute, since fast-dynamic voltage disturbances are not taken into account, similar to [21], [26].

The apparent power definition in the IEEE 1459-2010 Standard, [27], divides the apparent power to a fundamental component and a non-fundamental one. The latter can be divided into to three distinctive items:

$$S_N^2 = S_H^2 + D_I^2 + D_V^2 = \sum_{h\neq 1} V_h^2 \cdot \sum_{h\neq 1} I_h^2 + V_1^2 \cdot \sum_{h\neq 1} I_h^2 + I_1^2 \cdot \sum_{h\neq 1} V_h^2 \tag{9}$$

where the terms D_I and D_V imply the coupling between f and the HFs. The term S_H is the sum of all HFs' powers except f. It can be further expressed as:

$$S_H^2 = \sum_{h\neq 1} (V_h \cdot I_h)^2 = \sqrt{P_H^2 + D_H^2} \tag{10}$$

The term D_H refers to the distortion power of the combination of the HFVs and harmonic currents, thus, D_H includes again the coupling of different frequencies. The only term that genuinely expresses the power that is the product of voltage and current at the same HF is P_H, since it is caused by the HFV of order **h** and the component of the HF current of order h in-phase with the HFV of order h. Based on this analysis, P_H is going to be the power that is calculated at specific HF after the DHPF. The sensitivity matrix at $HF = h$ can be computed by inverting the Jacobian matrix calculated after the DHPF as:

$$\mathbf{S}_V^{\mathbf{h}} = \begin{bmatrix} \frac{\partial \theta^{\mathbf{h}}}{\partial \mathbf{P}^{\mathbf{h}}} & \frac{\partial \theta^{\mathbf{h}}}{\partial \mathbf{Q}^{\mathbf{h}}} \\ \frac{\partial |\bar{\mathbf{V}}^{\mathbf{h}}|}{\partial \mathbf{P}^{\mathbf{h}}} & \frac{\partial |\bar{\mathbf{V}}^{\mathbf{h}}|}{\partial \mathbf{Q}^{\mathbf{h}}} \end{bmatrix} \tag{11}$$

where $P_H = \mathbf{P}^{\mathbf{h}}$ and $\mathbf{V}^{\mathbf{h}}$ is the magnitude of the HFV. It is proposed that the harmonic active power sensitivity submatrix $\frac{\Delta V_i^{\mathbf{h}}}{dP_j^{\mathbf{h}}}$ is isolated and calculated for the nodes of interest within a LV DN. Similar to the analysis for the fundamental frequency, for a given HFV variation at a specific node i that contains a NLL, a CI-DRES at node j presenting a high $\frac{\Delta V_i^{\mathbf{h}}}{dP_j^{\mathbf{h}}}$ sensitivity will need to inject a smaller amount of low-order harmonic current at $HF = \mathbf{h}$ compared to a CI-DRES with a small sensitivity. Based on $\frac{\Delta V_i^{\mathbf{h}}}{dP_j^{\mathbf{h}}}$, the following procedure is proposed: For each harmonic the most critical CI-DRES node (CR-DRES) per harmonic is the one that has the maximum sum, similar to [25] for the fundamental frequency:

$$\text{CR-DRES}^{\mathbf{h}} = \mathbf{MAX} \left[\sum_{\forall j \in N} \frac{\partial V_i^{\mathbf{h}}}{\partial P_j^{\mathbf{h}}} \right] \quad \forall i \in N \tag{12}$$

This means that for a specific voltage variation, the smallest amount of $\partial P^{\mathbf{h}}$ will be provided by the CR-DRES$^{\mathbf{h}}$. This corresponds to the smallest amount of $|\bar{I}_{i,G}^{\mathbf{h}}|$ (the CI-DRES harmonic current injection at $HF = h$ and node i). It can be expected that the CR-DRES$^{\mathbf{h}}$ are different per harmonic. It should be noted that lower order HFVs have larger magnitude than higher HFVs. This means that in order to compensate the lower-order HFV, larger $|\bar{I}^{\mathbf{h}}_{i,G}|$ is needed to be provided, e.g., $|\bar{I}^{\mathbf{5}}_{i,G}| > |\bar{I}^{\mathbf{7}}_{i,G}|$. The CI-DRES thermal limit is calculated based on:

$$\frac{{P_i^G}^2 + {Q_i^G}^2}{3{V_i^f}^2} + \sum_{h \in HF} |\bar{I}^{\mathbf{h}}_{i,G}|^2 \leq {I^n_{i,G}}^2 \quad \forall i \in N \tag{13}$$

where $I^n_{i,G}$ denotes the rated current of the CI-DRES connected at node i. In the sum of harmonic currents, the largest share corresponds to the lowest order HF. For this reason, after computing all CR-DRES$^{\mathbf{h}}$ for $h = 5, 7, 11, 13$, it is suggested that the most critical CI-DRES node, **CR-DRES** is determined to be CR-DRES$^{\mathbf{5}}$, i.e., the one that corresponds to the lowest order HFV to be compensated. This **CR-DRES** is the first one to reach its thermal limits, hence, it will have the highest "burden" or contribution to the harmonic mitigation as an AS. The second one to exhaust the thermal capacity will be CR-DRES$^{\mathbf{7}}$, etc. With this process, in case there exist many NLLs within a LV DN, the DNO will be able to identify the CI-DRES that is most likely to provide harmonic mitigation most efficiently. In this way, the DNO could allow an oversized CI-DRES converter with respect to the CI-DRES primary source, so as to have more available capacity. The CI-DRES total daily contribution to the AF process can be quantified through the Fourier transform, as suggested in [28]:

$$D_{day} = \sqrt{\sum_{h=1}^{13} D^2_{h,day}} \tag{14}$$

$$D_{h,day} = \sum_{k=1}^{1440} \left[I_h(k) - I_1(k)\lambda_h \right], I_h(k) > I_1(k) \cdot \lambda_h \tag{15}$$

The actual contribution, $D_{h,day}$, of a DRES over a day (1440 min) for the mitigation of the *h-th* harmonic is given in (14): $I_1(k)$ and $I_h(k)$ are the 1-min rms value of the fundamental current and harmonic current h respectively injected by the CI-DRES during the k_{th} minute, and λ_h is the coefficient of individual HF, [17], e.g., $\lambda_h = 0.04$ for $h < 11$, $\lambda_h = 0.02$ for $11 \leq h < 17$, etc. Based on this analysis, it is expected that the **CR-DRES** will have the highest D_{day} value. The proposed framework can be used in two ways: (i) LV DN planning and CI-DRES sizing; (ii) development of a distributed harmonic mitigation control scheme based on the sensitivity analysis, similar to [25] regarding the active power curtailment or to [21] regarding the RP provision for voltage regulation.

III. Simulation Results

In this section the validity of the mathematical framework is assessed via simulations: (i) in the time domain (TD) via the PowerSim (PSIM) software package, where the virtual conductance method presented in [5] is applied; (ii) by performing DHPFs in OpenDSS. In both types of simulations two Cases under a modified version of the CIGRE European LV Residential DN of Fig. 1 are simulated - the cases are denoted in Fig. 1. The CI-DRES active and RP are set to $10kW$ and 0, respectively. In the TD simulations, the NLLs are modelled as a 3-phase six-pulse diode bridges with $P_{\text{NL}} = 40kW$

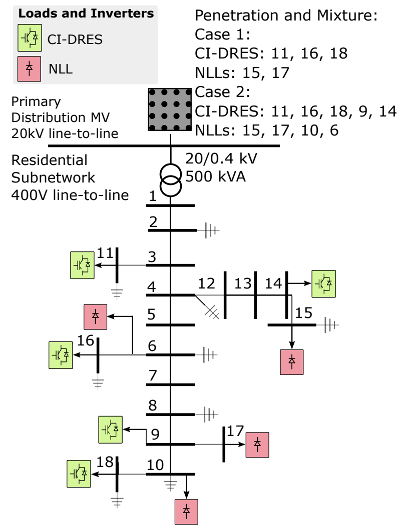

Fig. 1. Examined Topology of the CIGRE European LV Residential DN

TABLE I
6-pulse NLL Harmonic Current Spectrum

h	5	7	11	13
Magnitude, %	50	24	9.1	7.7

at $V_{\text{NL}} = 0.4kV$, while the DC output of the bridge is $C_{\text{NL}} = 1mF$ parallel to $R_{\text{NL}} = 7.05\Omega$. This NLL type has been chosen on purpose, because it causes high THD_V at $h = 5, 7, 11, 13$. The NLL distortion power is $34kVAr$ at V_{NL}. At V_{NL} the NLL harmonic spectrum at the examined frequencies is shown in Table I. This NLL has been designed, so that the distortion is intentionally larger in the 5^{th} and 7^{th} HFs than in the typical 6-pulse NLL proposed in [20] and used in [9], [10]. The CI-DRES are modelled via the control structure of [5]. The values of the current magnitudes and angles that were calculated by the TD simulations based on the local control of [5] are used as inputs in the DHFPs so as to evaluate the results in both simulation platforms.

In Fig. 2 a comparison between the TD simulations and the DHPFs under no control and with the control of [5] can be observed. In the scenarios noted as "No Control" the CI-DRES AF is de-activated, while the scenarios "With Control" all CI-DRES participate in the harmonic mitigation via the virtual conductance approach of [5]. In all HFs it can be noticed that: (a) TD profiles at the different HFs have larger values than the DHPFs, except at f. This is justified by the fact that in the DHPFs the slack bus voltage is zero and due to the frequency decoupling; (b) The previous condition leads to an almost "steady-state error" between the DHPFs and the TD simulations, however, the shape of the profiles is the same. This is especially important for the "No Control" cases where the sensitivity matrix is calculated; it means that the same results can be derived when using DHPFs instead of detailed models in the TD; (c) In all cases the HD_V is reduced with the control proposed in [5]. Note that there exist no such comparison between DHPFs and TD simulations in the technical literature.

Tables II-V show the results of the sensitivity analysis for Case 1. Tables VI-IX show the results of the sensitivity analysis for Case 2. The sensitivity matrices are calculated

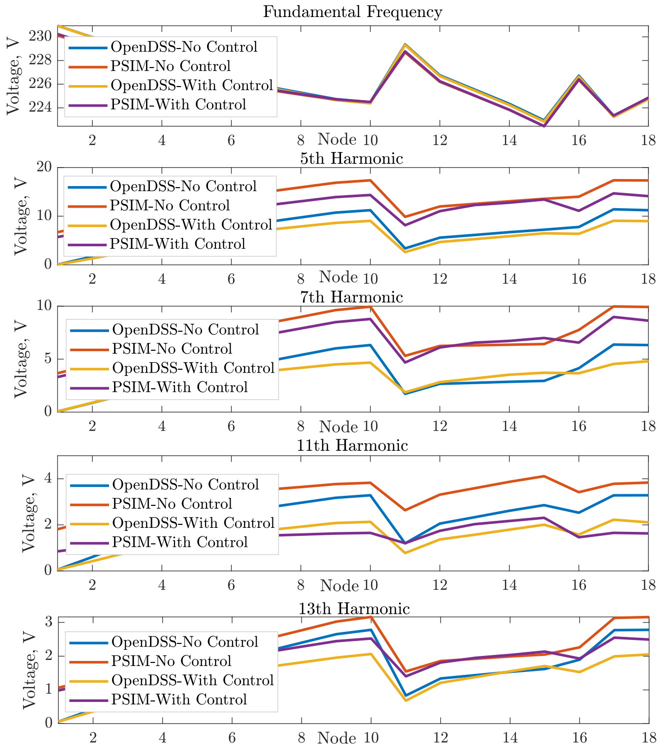

Fig. 2. Voltage Harmonic Distortion per node and HFs:Comparison between TD and DHPF

via (11) for each HF after performing DHPF under the "No Control" scenarios. In Fig. 3 the sum of the squares of the RMS currents (HFs $h = 5, 7, 11, 13$ and f using (13)) in the TD simulations are illustrated for Cases 1 and 2. Note that in [5] a tolerance zone was added to the control so as to ensure that the harmonic currents do not oscillate. Since the thermal limit of the CI-DRES is 18.1 A, the limit of the tolerance zone is approximately 17.8A. In Case 1 it can be derived based on (12) and Tables II-V that the critical CI-DRES at $h = 5$, CR-DRES$^{\mathbf{5}}$, is CI-DRES 18 (CI-DRES at Node 18 of Fig. 1); the critical CI-DRES at $h = 7$, CR-DRES$^{\mathbf{7}}$ is CI-DRES 18, the critical CI-DRES at $h = 11$, CR-DRES$^{\mathbf{11}}$ is CI-DRES 18, while the critical CI-DRES at $h = 13$, CR-DRES$^{\mathbf{13}}$ is CI-DRES 16. Thus, based on the proposed framework the most critical CI-DRES, **CR-DRES**, at all HFs is CI-DRES 18, while the 2^{nd} critical is CI-DRES 16 (Node 16 of Fig. 1). The validity of the proposed framework for Case 1 is demonstrated in Fig. 3-(Upper); It is evident that CI-DRES 18 is the 1^{st} to exhaust its thermal capacity, CI-DRES 16 is the 2^{nd}, while CI-DRES 11 is the last one. This is in accordance with the proposed framework. The same evaluation is performed for Case 2: it can be derived based on (12) and Tables VI-IX that the critical CI-DRES at $h = 5$, CR-DRES$^{\mathbf{5}}$ are both CI-DRES 18 **and** CI-DRES 9, CR-DRES$^{\mathbf{7}}$ is CI-DRES 18 (with a really slight difference from CI-DRES 9), CR-DRES$^{\mathbf{11}}$ is CI-DRES 16, while CR-DRES$^{\mathbf{13}}$ is CI-DRES 18 (with a really slight difference from CI-DRES 9). Based on the proposed framework the most critical CI-DRES, **CR-DRES**, at all HFs is CI-DRES 18, the 2^{nd} critical is CI-DRES 9 and the 3^{rd} critical is CI-DRES 16. The validity of the proposed framework for Case 2 is demonstrated in Fig. 3-

TABLE II
SENSITIVITY MATRIX FOR CASE 1 AND h=5: 3 CI-DRES/2 NLLS

	CI-DRES Node		
NLL Node	**11**	**16**	**18**
15	0.0052	0.0056	0.0059
17	0.0045	0.0043	0.0048

TABLE III
SENSITIVITY MATRIX FOR CASE 1 AND h=7: 3 CI-DRES/2 NLLS

	CI-DRES Node		
NLL Node	**11**	**16**	**18**
15	0.013	0.013	0.014
17	0.010	0.010	0.010

TABLE IV
SENSITIVITY MATRIX FOR CASE 1 AND h=11: 3 CI-DRES/2 NLLS

	CI-DRES Node		
NLL Node	**11**	**16**	**18**
15	0.133	0.142	0.145
17	0.111	0.115	0.116

TABLE V
SENSITIVITY MATRIX FOR CASE 1 AND h=13: 3 CI-DRES/2 NLLS

	CI-DRES Node		
NLL Node	**11**	**16**	**18**
15	0.124	0.130	0.132
17	0.145	0.145	0.142

TABLE VI
SENSITIVITY MATRIX FOR CASE 2 AND h=5: 5 CI-DRES/4 NLLS

	CI-DRES Node				
NLL Node	**9**	**11**	**14**	**16**	**18**
6	0.0056	0.0052	0.0049	0.0051	0.0057
10	0.0036	0.0040	0.0037	0.0039	0.0034
15	0.0059	0.0052	0.0016	0.0057	0.0060
17	0.0036	0.0040	0.0037	0.0039	0.0037

TABLE VII
SENSITIVITY MATRIX FOR CASE 2 AND h=7: 5 CI-DRES/4 NLLS

	CI-DRES Node				
NLL Node	**9**	**11**	**14**	**16**	**18**
6	0.0265	0.0250	0.0246	0.0253	0.0266
10	0.0185	0.0186	0.0183	0.0188	0.0182
15	0.0297	0.0273	0.0105	0.0290	0.0297
17	0.0184	0.0185	0.0182	0.0187	0.0186

TABLE VIII
SENSITIVITY MATRIX FOR CASE 2 AND h=11: 5 CI-DRES/4 NLLS

	CI-DRES Node				
NLL Node	**9**	**11**	**14**	**16**	**18**
6	0.0618	0.0608	0.0592	0.0605	0.0620
10	0.0477	0.0494	0.0481	0.0492	0.0471
15	0.0580	0.0559	0.0377	0.0575	0.0581
17	0.0478	0.0494	0.0482	0.0492	0.0481

TABLE IX
SENSITIVITY MATRIX FOR CASE 2 AND h=13: 5 CI-DRES/4 NLLS

	CI-DRES Node				
NLL Node	**9**	**11**	**14**	**16**	**18**
6	0.156	0.151	0.150	0.153	0.156
10	0.116	0.114	0.114	0.116	0.115
15	0.156	0.150	0.120	0.155	0.157
17	0.116	0.115	0.114	0.117	0.117

(Lower): CI-DRES 18 and CI-DRES 9 almost simultaneously are the 1^{st} ones to exhaust their thermal capacity, CI-DRES 16 is the 2^{nd} one, CI-DRES 11 3^{rd} and CI-DRES 14 is the last one. These results are also in accordance with the proposed framework. Hence, the sensitivity analysis can be used either for the evaluation of decentralized APF methods and the CI-DRES contribution per harmonic.

IV. CONCLUSIONS AND FUTURE RESEARCH

In this paper the mathematical framework for the harmonic sensitivity analysis is performed in order to evaluate the

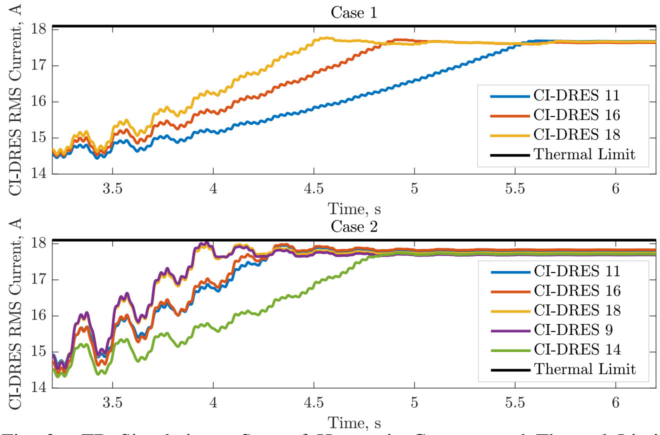

Fig. 3. TD Simulation - Sum of Harmonic Currents and Thermal Limit: (Upper):Case 1, (Lower):Case 2

contribution of each CI-DRES in the decentralized harmonic filtering process, with the presence of NLLs within the DN based on the CI-DRES location and available capacity. The available capacity is an important parameter, since in the future, the CI-DRES might provide simultaneously several services, e.g. reactive power or virtual inertia. The mathematical framework for the quantification of the CI-DRES contribution is validated via simulations in the time domain and via harmonic power flows in the benchmark European CIGRE LV DN under various mixtures of CI-DRES/NLLs. The derived conclusions will pave the path, so that a new distributed harmonic mitigation algorithm is developed based on the sensitivity matrix at each harmonic. This framework could also be used by the DNOs for CI-DRES sizing during the planning stage if considering the harmonic mitigation process. Finally, based on the developed framework, the active filtering can be treated as a new ancillary service to be provided by CI-DRES at DN level and introduced in respective markets.

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